Chapter 2 - Day 2

If we want to compute the Instantaneous Rate of Change at a point $x=a$, then we usually compute the ARoC between $x=a$ and $x=a+h$ where we think of h as being a small number.

then $$\text{AROC} = \frac{f(a+h)-f(a)}{(a+h)-a} = \frac{f(a+h)-f(a)}{h}$$

We then let h get closer and closer to 0, we find a limit.

$$v(a) = \lim_{h \to 0} \frac{f(a+h)-f(a)}{h}$$

IRoC at x .... compute AROC from x to x+h then let $h \to 0$

Ex: a car travels along a straight line with position given by $f(x) = 9x^2 + 1$.

a) find the velocity when $x = 3$ seconds.

* find IRoC from $x=3$ to $x=3+h$

$$\text{ARoC} = \frac{f(3+h) - f(3)}{(3+h) - 3} = \frac{[9(3+h)^2 + 1] - [9(3^2)+1]}{h}$$

$$= \frac{9(9+6h+h^2)+1 - 82}{h}$$

$$= \frac{\cancel{81} + 54h + 9h^2 \cancel{+1} \cancel{-82}}{h}$$

$$= \frac{54h + 9h^2}{h} = \frac{\cancel{h}(54+9h)}{\cancel{h}}$$

$$= 54 + 9h$$

* now let $h \to 0$

$$54 + 9(0) = \boxed{54}$$

b) find the velocity when $x = t$ seconds.

$$AROC = \frac{f(t+h) - f(t)}{(t+h) - t} = \frac{[9(t+h)^2 + 1] - [9t^2 + 1]}{h}$$

$$= \frac{9(t^2 + 2th + h^2) + 1 - 9t^2 - 1}{h}$$

$$= \frac{\cancel{9t^2} + 18th + 9h^2 \cancel{+1} \cancel{-9t^2} \cancel{-1}}{h}$$

$$= \frac{\cancel{h}(18t + 9h)}{\cancel{h}} = 18t + 9h$$

let $h \to 0$

$$18t + 9(0) = 18t$$

so $v(t) = 18t$

Ex: let $g(k) = k^2 + 4k + 9$

a) find the IROC as a function of k.

* find AROC from k+h to k

$$AROC = \frac{g(k+h) - g(k)}{(k+h) - k}$$

$$= \frac{[(k+h)^2 + 4(k+h) + 9] - [k^2 + 4k + 9]}{h}$$

$$= \frac{\cancel{k^2} + 2kh + h^2 + \cancel{4k} + 4h + \cancel{9} - \cancel{k^2} - \cancel{4k} - \cancel{9}}{h}$$

$$= \frac{2kh + h^2 + 4h}{h} = \frac{\cancel{h}(2k + h + 4)}{\cancel{h}}$$

$$= 2k + h + 4$$

* IROC let $h \to 0$

$$2k + (0) + 4 = \boxed{2k + 4 = IROC}$$

b) find the IROC at k=1.

$$IROC = 2(1) + 4 = \boxed{6}$$

the derivative of $f(x)$ at x,

denoted $f'(x)$ is

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

* looks like IRoC!

other notations:

$$f'(x) = \lim_{\Delta x \to 0} \frac{\Delta f}{\Delta x} = \frac{df}{dx}$$

or if $y = f(x)$ then

$$y' = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \frac{dy}{dx}$$

$\underline{\text{Ex}}$: let $f(x) = mx+b$. Show $f'(x) = m$.

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{[m(x+h)+b]-[mx+b]}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{mx}+mh\cancel{+b}\cancel{-mx}\cancel{-b}}{h}$$

$$= \lim_{h\to 0} \frac{m\cancel{h}}{\cancel{h}} = \lim_{h\to 0} m = m$$

$\underline{\text{Ex}}$: let $f(x) = ax^2+bx+c$. Show $f'(x) = 2ax+b$.

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

$$= \lim_{h\to 0} \frac{[a(x+h)^2+b(x+h)+c]-[ax^2+bx+c]}{h}$$

$$= \lim_{h\to 0} \frac{a(x^2+2xh+h^2)+bx+bh+c-ax^2-bx-c}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{ax^2}+2axh+ah^2+\cancel{bx}+bh+\cancel{c}\cancel{-ax^2}\cancel{-bx}\cancel{-c}}{h}$$

$$= \lim_{h\to 0} \frac{2axh+ah^2+bh}{h}$$

$$= \lim_{h\to 0} 2ax+ah+b = 2ax+a(0)+b$$

$$= 2ax+b \checkmark$$